

$f(x) = \frac{\sqrt{x^2 + 12} - 2}{x^2}$ determine the following.

1) $f(2) = \frac{2}{4} = \frac{1}{2}$

2) $f(0) = \frac{\sqrt{12} - 2}{0} = \emptyset$

Find out what is happening to the graph of a function when we cannot evaluate it at a certain value. We use a limit.

Instead of asking what does $f(x)$ equal when x is a certain value, we ask *what does $f(x)$ approach as x approaches a certain value.*

Limit Notation

$$\lim_{x \rightarrow a} f(x) = L$$

One Sided Limits

Left Hand Side:

$$\lim_{x \rightarrow a^-} f(x)$$

Right Hand Side:

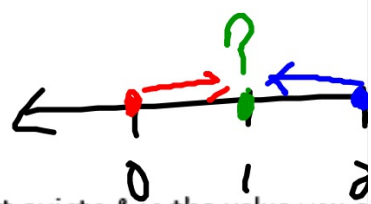
$$\lim_{x \rightarrow a^+} f(x)$$

For a limit to exist:

The left hand side must equal the right hand

Example 2: Find the following limit using a table of values:

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} = \frac{\sqrt[3]{1} - 1}{\sqrt{1} - 1} = \frac{1 - 1}{1 - 1} = \frac{0}{0}$$



First, find the left & right hand side limits... if they are equal then the limit exists & is the value you get

$$\lim_{x \rightarrow 1^-} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} =$$

x	0	.5	.9	.99
f(x)	1	.7044	.6729	.6672

$$\lim_{x \rightarrow 1^+} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} =$$

x	2	1.5	1.1	1.01
f(x)	.6275	.6439	.6613	.6661

$\frac{2}{3}$

You can also find limits by looking at the graph of a function.
Given the graph of $f(x)$, find the following limits if they exist.

a) $\lim_{x \rightarrow 1^-} f(x) = 1$

b) $\lim_{x \rightarrow 1^+} f(x) = 3$

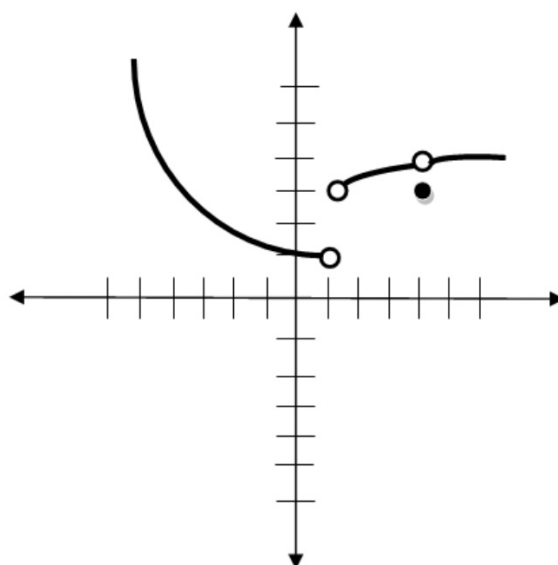
c) $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

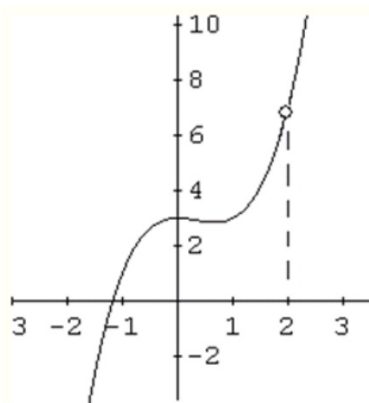
d) $\lim_{x \rightarrow 4^-} f(x) = 4$

e) $\lim_{x \rightarrow 4^+} f(x) = 4$

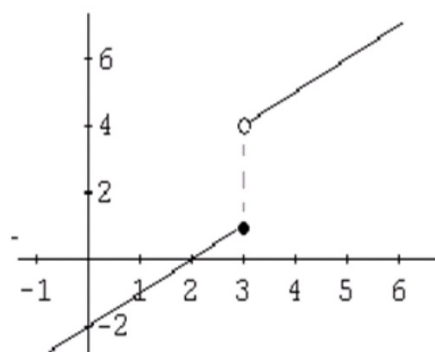
f) $\lim_{x \rightarrow 4} f(x) = 4$

g) $f(4) = 3$

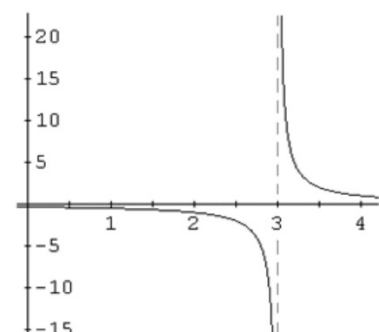




$$\begin{aligned}\lim_{x \rightarrow 2^-} f(x) &= 7 \\ \lim_{x \rightarrow 2^+} f(x) &= 7 \\ \lim_{x \rightarrow 2} f(x) &= 7 \\ f(2) &= \text{DNE}\end{aligned}$$



$$\begin{aligned}\lim_{x \rightarrow 3^-} g(x) &= 4 \\ \lim_{x \rightarrow 3^+} g(x) &= \text{DNE} \\ \lim_{x \rightarrow 3} g(x) &= \text{DNE} \\ g(3) &= 1\end{aligned}$$



$$\begin{aligned}\lim_{x \rightarrow 3^-} h(x) &= -\infty \\ \lim_{x \rightarrow 3^+} h(x) &= \infty \\ \lim_{x \rightarrow 3} h(x) &= \text{DNE} \\ h(3) &= \text{DNE}\end{aligned}$$

If a graph is continuous you can simply plug in the value to find the limit.

1) $\lim_{x \rightarrow 2} 2x + 3x =$

2) $\lim_{x \rightarrow -3} x^2 - 5 =$

3) $\lim_{\theta \rightarrow \pi} \sin \theta =$

$$4) \lim_{x \rightarrow 4} \frac{3}{x + 2}$$

$$5) \lim_{x \rightarrow \frac{1}{2}} (3x^2 - 2) =$$

$$6) \lim_{\theta \rightarrow \frac{5\pi}{4}} \tan \theta =$$

The indeterminate form:

When direct substitution gives you $\frac{0}{0}$, simplify the function algebraically and substitution again.

Algebraic methods for finding limits:

1. Direct substitution

2. Simplify expressions

$$7) \lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} =$$

$$8) \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1} =$$

$$9) \lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{x^2 + 8x - 48} =$$

How to find a limit

